2020

(Held in 2021)

PHYSICS

Paper : CC–5

(Mathematical Physics—II)

Full Marks : 60

Time : 3 hours

The figures in the margin indicate full marks for the questions

1. Choose the correct answer from the following questions : 1×5=5

(a) The Fourier series of the even function

(i) contains only cosine terms

(ii) contains only sine terms

(iii) contains both cosine and sine terms

(iv) does not contain cosine and sine terms

(b) Truncation error arises due to

(i) limitation of mathematical tables or digital computers

(ii) rounding off of the numbers

(iii) use of approximate results in an infinite series

(iv) All of the above

(c) Which of the following is not Dirichlet's condition for the Fourier series expansion?

(i) $f(x)$ is periodic, single-valued, finite

(ii) $f(x)$ has finite number of maxima and minima

(iii) $f(x)$ is aperiodic, single-valued, finite

(iv) $f(x)$ has finite number of discontinuities in only one period

(d) Given that

$$\Gamma\left(\frac{1}{n}\right)\Gamma\left(\frac{2}{n}\right)\Gamma\left(\frac{3}{n}\right)\,.....\,\Gamma\left(\frac{n-1}{n}\right)=\frac{(2\pi)^{\frac{n-1}{2}}}{\sqrt{n}},$$

the value of

$$\Gamma(0\cdot1)\;\Gamma(0\cdot2)\;\Gamma(0\cdot3)\;.....\;\Gamma(0\cdot9)$$

is

(i) 0

(ii) $\dfrac{(2\pi)^{\frac{9}{2}}}{\sqrt{10}}$

(iii) 1

(iv) $\dfrac{(2)^{\frac{9}{2}}}{\sqrt{10\pi}}$

(e) Solutions of Laplace's equation are

(i) harmonic functions

(ii) analytic within the domain where the equation is satisfied

(iii) Both *(i)* and *(ii)* are correct

(iv) Both *(i)* and *(ii)* are wrong

2. Answer the following questions : 2×5=10

(a) If $R = \frac{4xy^2}{z^3}$ and errors in x, y, z be 0·001 each, then calculate the maximum relative error while measuring R at $x = y = z = 1$.

(b) Using the properties of gamma function, show that $\Gamma\left(\frac{1}{2}\right) = \sqrt{\pi}$.

(c) Consider any complex periodic, piecewise continuous function

$$f(x) = a_0 + \sum_{n=1}^{\infty} a_n \cos nx + \sum_{n=1}^{\infty} b_n \sin nx$$

Integrate this function between the limits $(-\pi, \pi)$ to show that

$$a_0 = \frac{1}{2\pi} \int_{-\pi}^{+\pi} f(x)\, dx$$

(d) Consider the standard form of the linear homogeneous differential equation

$$\frac{d^2y}{dx^2} + P(x)\frac{dy}{dx} + Q(x)y = 0$$

and restrict yourself to a solution in the neighbourhood of $x = 0$. Under what circumstances the point $x = a$ will be called *(i)* an ordinary point *(ii)* a regular point?

(e) Use the method of separation of variables, to find the solution of the partial differential equation

$$3\frac{\partial u}{\partial x} + 2\frac{\partial u}{\partial y} = 0$$

3. Answer any *five* of the following questions : 5×5=25

(a) Find the Fourier series of the function e^x in the interval $-\pi < x < \pi$.

(b) Prove the following orthogonal property of the Legendre's polynomials :

$$\int_{-1}^{+1} P_m(x)\, P_n(x)\, dx = 0; \qquad m \neq n$$

(c) Write briefly about the following partial differential equations frequently encountered in physics :

(i) Laplace's equation

(ii) Poisson's equation

Also mention at least one situation where each of the two may be applicable. 2+2+1=5

(d) John found the hours of sunshine *vs* the number of ice creams sold at the shop from Monday to Friday :

Hours of sunshine (x)	*Ice creams sold* (y)
2	4
3	5
5	7
7	10
9	15

Use the method of least squares to find the best value of m and b that suits the data :

$$y = mx + b$$

On Saturday morning, John hears the weather forecast which says "We expect 8 hours of sun today." How many ice creams should he make on that day to sell?

(e) The beta and gamma functions are defined as

$$\beta(m,n) = \int_0^1 x^{m-1}(1-x)^{n-1}\,dx; \quad m > 0,\ n > 0$$

$$\Gamma(n) = \int_0^\infty e^{-x} x^{n-1}\,dx; \ n > 0$$

Show that the two are related by the equation

$$\beta(m,n) = \frac{\Gamma(m)\,\Gamma(n)}{\Gamma(m+n)}$$

(f) Starting from the generating function of Hermite polynomials, deduce the Rodrigue's formula

$$H_n(x) = (-1)^n e^{x^2} \frac{d^n}{dx^n}\left(e^{-x^2}\right)$$

Hence find the values of $H_0(x)$ and $H_1(x)$. 3+1+1=5

(g) Prove the following recurrence relations : 2½+2½=5

(i) $nP_n = (2n-1)xP_{n-1} - (n-1)P_{n-1}$

(ii) $P'_{n+1} - xP'_n = (n+1)P_n$

4. Answer any *two* of the following questions : 10×2=20

(a) Find the power series solution of the linear oscillation equation

$$\frac{d^2y}{dx^2} + \omega^2 y = 0$$

in powers of x.

(b) Write down Laplace's equation in three-dimensional Cartesian coordinate system and find the solution of the same using the method of separation of variables. 2+8=10

(c) Mention some uses of Fourier series. Apply Fourier's theorem to analyze the output wave from a full wave rectifier when input wave is of the form

$$E = E_0 \sin \omega t$$ 2+8=10

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