

4. Answer any two of the following questions :

10×2=20

- (a) Obtain the power series solution of the following equation :

$$(1-x^2)\frac{d^2y}{dx^2} - 2x\frac{dy}{dx} + n(n+1)y = 0$$

- (b) Prove that

$$\int_{-\infty}^{+\infty} e^{-x^2} H_n(x) H_m(x) dx = \begin{cases} 0 & , \text{ if } m \neq n \\ \sqrt{\pi} \cdot 2^n n! & , \text{ if } m = n \end{cases}$$

- (c) Determine the solution of one-dimensional heat equation

$$\frac{\partial u}{\partial t} = c^2 \frac{\partial^2 u}{\partial x^2}$$

subject to the boundary conditions $u(0, t) = 0$, $u(L, t) = 0$ ($t > 0$) and the initial condition $u(x, 0) = x$, L being the length of the bar.

2021

(Held in 2022)

PHYSICS

Paper : CC-5

(Mathematical Physics—II)

Full Marks : 60

Time : 3 hours

The figures in the margin indicate full marks for the questions

1. Choose the correct answer from the following questions : 1×5=5

(a) $\int_0^{2\pi} \sin nx \, dx =$

(i) π (ii) ∞

(iii) 0

(iv) None of the above

(2)

(b) In Fourier series, the function is said even if

(i) $f(-x) = f(x)$

(ii) $f(-x) = -f(x)$

(iii) $f(-x) = 0$

(iv) None of the above

(c) The regular singular point of the differential equation

$$2x^2 \frac{d^2y}{dx^2} + 3x \frac{dy}{dx} + (x^2 - 4)y = 0$$

is

(i) $x = 0$

(ii) $x = 1$

(iii) $x = -1$

(iv) None of the above

(d) $\Gamma\left(\frac{1}{2}\right) = ?$

(i) 0

(ii) $\sqrt{\pi}$

(iii) 1

(iv) None of the above

(3)

(e) Stochastic errors can be minimised

(i) by taking large number of observations

(ii) by calculating the average of large number of observations

(iii) by measuring the instrumental error

(iv) None of the above

2. Answer the following questions : 2×5=10

(a) Verify if Frobenius method is applicable to the following equation :

$$\frac{d^2y}{dx^2} - \frac{6y}{x^3} = 0$$

(b) Write two Dirichlet's conditions for Fourier series.

(c) Show that

$$\beta(m, n) = \beta(n, m)$$

(d) Find the value of $\Gamma\left(\frac{3}{2}\right)$.

(e) For simple pendulum

$$T = 2\pi\sqrt{\frac{l}{g}}$$

$$l = 92.95 \pm 0.2 \text{ cm}$$

$$T = 1.936 \pm 0.003 \text{ sec}$$

Find the error for g .

3. Answer any five from the following questions :

$$5 \times 5 = 25$$

(a) Find the Fourier series

$$f(x) = x; 0 < x < 2\pi$$

(b) Find the regular singular point of the given equation and also find the indicial equation

$$\frac{d^2y}{dx^2} + y = 0 \quad 2+3=5$$

(c) Find the recurrence relation of the following equation :

$$y'' - 2xy' + 2\lambda y = 0$$

λ is constant.

(d) Find $H_1(x)$ and $H_2(x)$ from Hermite

polynomial $H_n(x)$: $2^{1/2} + 2^{1/2} = 5$

$$H_n(x) = \sum_{r=0}^n (-1)^r \frac{n! (2x)^{n-2r}}{r! (n-2r)!}$$

$$N = \frac{n}{2} \quad \text{if } n \text{ is even}$$

$$N = \frac{(n-1)}{2} \quad \text{if } n \text{ is odd}$$

(e) Evaluate using β and Γ functions :

$$\int_0^1 x^4 (1-\sqrt{x})^5 dx$$

(f) Solve using the method of variable separation

$$\frac{\partial u}{\partial x} + u = \frac{\partial u}{\partial t}$$

where $u(x, 0) = 4e^{-3x}$.

(g) What is standard error? Write the various steps to find the standard error and hence find the standard error from the following data :

The height of five persons are as follows :

150.5 cm, 170.5 cm, 160.0 cm,

161.0 cm, 170.0 cm $1+2+2=5$